Estimation of maintenance efficiency in imperfect repair models

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Abstract

The aim of that paper is to study the estimation of maintenance efficiency in two imperfect repair models, called Arithmetic Reduction of Intensity or Age model, both in the case of finite memory. These models have been proposed by Doyen and Gaudoin (2004) and include some virtual age models among which the one proposed by Kijima, Morimura and Suzuki (1998). First, the asymptotic almost sure behavior of the failure and cumulative failure intensities of those models is studied. The almost sure convergence and asymptotic normality of several estimators (including maximum likelihood) of repair efficiency are derived, when the wear out process without repair is known. Finally, results issued from the empirical study of the coverage rate of confidence intervals for repair efficiency are given.

Introduction

All important industrial systems (like nuclear power plants, planes, trains) are subjected to corrective maintenance actions or repairs that are supposed to reduce the frequency of occurrence of system failures. The assessment of the efficiency of these repairs is of great practical interest, but it has been seldom studied. Stochastic modelling of the failure and repair process is usually done with random point processes. Let $\{T_i\}_{i\geq 1}$ be the successive failure times of a repairable system, starting from $T_0 = 0$. Let N_t be the number of failures observed up to time t. The repair times are assumed to be negligible or not taken into account. Then, the failure process is defined equivalently by the random processes $\{T_i\}_{i\geq 1}$ or $\{N_t\}_{t\geq 0}$. The distribution of these processes is completely given by the failure intensity, defined as:

$$\forall t \geq 0, \quad \lambda_t = \lim_{dt\to 0} \frac{1}{dt} P(N_{t+dt} - N_{t^-} = 1|\mathcal{H}_{t^-})$$

where N_{t^-} is the left hand limit of N_t, $\mathcal{H}_t$ is the sigma-algebra generated by all the failure times that take place up to time t and $\mathcal{H}_{t^-} = \cup_{s<t}\mathcal{H}_s$.

1 Imperfect repair models

The basic assumptions on maintenance efficiency are known as minimal repair or As Bad As Old (ABAO) and perfect repair or As Good As New (AGAN). In the ABAO case, each maintenance is supposed to leave the system in the same state as it was before failure. The corresponding random processes are the non homogeneous Poisson processes. These processes are such that the failure intensity is only a function of time:

$$\forall t \geq 0, \quad \lambda_t = \lambda(t)$$

In the AGAN case, each maintenance perfectly repairs the system and leaves it as if it were new. The corresponding random processes are the renewal processes. These processes are such that the failure intensity is defined as:

$$\forall t \geq 0, \quad \lambda_t = \lambda(t - T_{N_{t^-}})$$

Obviously, reality is between these two extreme cases: standard repair reduces failure intensity but does not leave the system as good as new. This is known as imperfect repair or better than minimal repair.

Several models with that type of assumption have already been proposed (see for example a review in Pham and Wang 1996). One of the most famous is the Brown and Proschan (1983) model, in which system state after repair is AGAN with probability p and ABAO with probability $1-p$. Another very important class of models is the virtual age models proposed by Kijima (1989). Even if imperfect repair models have been proposed, only a few of them have been statistically studied, especially in what concerned the estimation of repair efficiency. Some authors like Shin, Lim and Lie (1996), Yun and Choung (1999), or Kaminskiy and Krivtsov (2000) have proposed simulation results for the properties of maximum likelihood parameters in some particular virtual age models. But no theoretical results have been proved.

This paper will be focused on the study of the properties of two classes of models known as Arithmetic Reduction of Age (ARA) and Arithmetic Reduction of Intensity (ARI) models (Doyen and Gaudoin 2004). The leading assumption of ARI models is to consider that each repair action reduces the failure intensity of an amount depending only of the past of the failure process. Then an ARI model with memory $m \geq 1$ (ARI_m) has a failure intensity defined as:

$$\lambda_t = \lambda(t) - \rho \sum_{j=0}^{Min(m-1, N_{t^-}-1)} (1-\rho)^j \lambda(T_{N_{t^-}-j})$$

where $\lambda(t)$ is the (deterministic) initial failure intensity (before the first failure). There exists interesting particular cases of this class of models. The ARI_∞ model supposes that repair reduces failure intensity of an amount proportional to the current value of the intensity. It leads to:

$$\lambda_t = \lambda(t) - \rho \sum_{j=0}^{N_{t^-}-1} (1-\rho)^j \lambda(T_{N_{t^-}-j})$$

The ARI_1 model considers that repair actions cannot reduce the global wear of the system, but only the relative wear since the last repair. The corresponding intensity is particularly simple:

$$\lambda_t = \lambda(t) - \rho\lambda(T_{N_{t^-}})$$

The principle of the ARA class of models is to consider that repair rejuvenates the system such that its intensity at time t is equal to the initial intensity at time A_t, where $A_t \leq t$. The properties of A_t are the same as those of λ_t in ARI models when the initial intensity is $\lambda(t) = t$. Then, by analogy with ARI models, we can build ARA models. And the failure intensity of an ARA_m model is:

$$\lambda_t = \lambda(t - \rho \sum_{j=0}^{Min(m-1, N_{t^-}-1)} (1-\rho)^j T_{N_{t^-}-j})$$

The ARA_1 model has a particularly simple failure intensity: $\lambda_t = \lambda(t - \rho T_{N_{t^-}})$ and appears to be the same as the Kijima, Morimura and Suzuki (1988) model. In ARI and ARA models, repair efficiency is characterized by a single parameter ρ:

- $0 < \rho < 1$: imperfect but efficient repair
- $\rho = 1$: optimal repair (perfect for ARA models)
- $\rho = 0$: minimal or useless repair
- $\rho < 0$: harmful repair

Then, assessing repair efficiency in these models is estimating parameter ρ.

2 Asymptotic behavior of the failure process

ARI_m and ARA_m models with finite memory have an interesting property: there exists a so-called asymptotic intensity. That is to say, the failure intensity has the same asymptotic behavior as the asymptotic intensity, λ_∞, equals for ARI_m models to $\lambda_\infty(t) = (1-\rho)^m\lambda(t)$ and for ARA_m models to $\lambda_\infty(t) = \lambda((1-\rho)^m t)$. This property is true when the three following assumptions are verified :

- **A1:** $\rho < 1$,
- **A2**: $\lambda(t) \to +\infty$,
- **A3**: $\lambda(t) - \lambda(t + o(1)) = o(\lambda(t))$,

The proof of this result and of the following properties is done thanks to the martingale results of Andersen and Co (1993) and Cocozza-Thivent (1997).

Property 1 *For an ARI_m or ARA_m model with finite memory and under assumptions A1 to A3, the failure intensity verifies:*

$$\lambda_t \overset{a.s.}{=} \lambda_\infty(t) + o(\lambda_\infty(t))$$

Assumptions A2 and A3 are in particular verified with strictly increasing power functions for the initial intensity:

- **A4**: $\lambda(t) = \alpha\beta t^{\beta-1}$, $\alpha > 0$ $\beta > 1$.

In addition, the difference between the failure intensity and the asymptotic intensity can be expressed.

Property 2 *For an ARI_m model with finite memory and under assumptions A1 to A3, or for ARA_m model with finite memory under assumptions A1 and A4, the cumulative failure intensity verifies:*

$$\Lambda_t \overset{a.s.}{=} \Lambda_\infty(s) + \frac{1-(1+m\rho)(1-\rho)^m}{\rho(1-\rho)^m} ln(\lambda(t)) + o(ln(\lambda(t))) \quad (1)$$

3 Estimation of repair efficiency

From these properties of the failure process, the almost sure convergence and asymptotic normality of some estimators of repair efficiency can be derived. The convergence and normality are proved in the case where the initial intensity is supposed to be known. Then, only parameter ρ has to be estimated. Let $\hat{\rho}_t^{MLE}$ be the maximum likelihood estimator (MLE) of ρ for an observation of the failure process over $[0,t]$ and let us assume that the MLE is researched in a known closed and bounded interval:

- $\tilde{\mathbf{A}}\mathbf{1}$: The true value of the repair efficiency parameter is in a known interval $[\rho_1, \rho_2]$ such that $-\infty < \rho_1 < \rho_2 < 1$. The MLE is searched in that interval.

Property 3 . *For an ARI_m model with finite memory and under assumptions $\tilde{A}1$, A2 and A3, or for ARA_m model with finite memory under assumptions $\tilde{A}1$ and A4, the maximum likelihood estimator of repair efficiency parameter verifies, for a single observation of the failure process over $[0,t]$:*

$$\forall \epsilon > 0, \qquad \left|\rho_0 - \hat{\rho}_t^{ML}\right| \Lambda(t)^{0.5-\epsilon} \xrightarrow[t\to+\infty]{a.s.} 0$$

$$\sqrt{\frac{\Lambda(t)}{(1-\rho_0)^m}} \left[(1-\rho_0)^m - (1-\hat{\rho}_t^{ML})^m\right] \xrightarrow[t\to+\infty]{\mathcal{L}} \mathcal{N}(0,1)$$

There exists explicit estimators (EE) having the same asymptotic properties with less restrictive assumptions.

Property 4 *For an ARI_m model with finite memory and under assumptions A1 to A3, or for ARA_m model with finite memory under assumptions A1 and A4, the explicit estimators:*

$$\hat{\rho}_t^{EI} = 1 - \left(\frac{N_t}{\Lambda(t)} \right)^{\frac{1}{m}}, \qquad \textit{for } ARI_m \textit{ models}$$

$$\hat{\rho}_t^{EA} = 1 - \left(\frac{N_t}{\Lambda(t)} \right)^{\frac{1}{m(\beta-1)}}, \qquad \textit{for } ARA_m \textit{ models}$$

$$\hat{\rho}_t^{EI2} = \frac{\Lambda(t) - N_t}{\int_0^t \lambda(T_{N_s})\, ds}, \qquad \textit{for } ARI_1 \textit{ model}$$

verify the same asymptotic properties as the MLE of property 3.

Thanks to the asymptotic normality of all that estimators, asymptotic confidence intervals for ρ can easily be deduced. Results issue from the empirical study of the coverage rate of these intervals will be developed in the oral presentation.

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